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THE MATHEMATICS TEACHER

EDITED BY
W. H. METZLER

ASSOCIATED WITH
EUGENE R. SMITH MAURICE J. BABB

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EDUCATIONAL VALUES IN MATHEMATICAL TEACHING.

By H. E. Hawkes.

(Continued from last issue.)

means to an end. It is the lever with which the windows of the mind are pried open, and we teachers must be content to see it discarded when this all-important work is done.

In the present discussion I am not thinking of the needs of the student who will use mathematics professionally. The requirements of the engineer and of the mathematician are comparatively simple and easy to satisfy. It is the general student, who, perhaps, has no idea of even going to college whom I have in mind. The kind of boy who will never use any fact of algebra or geometry more than once or twice in his life, and is then so surprised that he seeks out his old teacher, and tells him all about it—just to make him feel that he has been of some use in the world.

Many of the live questions of mathematical teaching take on new meaning when looked at from the point of view of transferability. I have time to mention but a single one, the problem of the correlation of the various mathematics subjects.

I am certain that all of us have had the chilling experience of asking a question of an algebra class which involves a little geometry. One boy spoke for many when he said: "I am certain that I could answer that question if I were only reciting in geometry." This merely means that the boy cannot transfer easily even from one class room in mathematics to another in the same department. That is, students are very far from having their complete mathematical knowledge available all of the time. We wish they might. Theoretically it seems very reasonable to suppose that if the hard and fast divisions into distinct subjects were removed, this difficulty of transference would not exist. For it is certain that there is sluggish flow of ideas not only between the subjects of algebra and geometry, but from instructor to instructor, and from class room to class room. So far as this one principle of transferability is concerned, its weight seems to be entirely on the side of a closer correlation of the different mathematical subjects; for such correlation would remove some of the barriers over which transference is difficult.

At many points pertinent and natural relations between algebra and geometry should be emphasized. Even if the boy is in his first year in algebra certain facts from geometry and physics may be stated, and the algebra of these facts worked out with telling effect. And in geometry use should be made of algebra on every reasonable occasion. The fundamental facts of trigonometry may very well be introduced in connection with the work on similar triangles in geometry, and many bridges between analytical geometry and advanced algebra should be constructed. All of this tends to make the line of division between the subjects less sharp and formidable.

But it does not mean that algebra and geometry should be completely amalgamated, or fused, as some of the exponents of the correlation movement call it—confused I should say. The principle of transferability is not the only one which comes into this discussion. The type of imagination, of reasoning, of mental asmosphere which is effective in geometry is quite different from that required in algebra, and I believe that both subjects would suffer from this fusion.

I will not go into details regarding the elements in the second category, that of mental processes, though there is ample material for discussion. But I must remark in passing that the educational ideal of our fathers which they tried to express under the nebulous name of "discipline" really includes much that the present analysis places in the second and the third category. Compare for educational content this ideal with that of some of us moderns who make the fact, the vocational fact, the ideal of our effort. In spite of its chill and formal sound, the older ideal is vastly deeper and broader and fuller than the modern one.

During the remaining moments let me emphasize some of the moral qualities which mathematics may be expected to evoke, and which are of transcendent educational importance, not only on account of their mathematical value, but on account of their intimate relation to the broad intellectual life of every serious man.

I will mention but three: The respect for truth, the use of the intuition, the appreciation of unity and harmony.

First, the respect for truth. In our subject we draw our conclusions from definitions and postulates which we lay down as absolute, not relative, and the theorems follow from these principles with a certainty which leaves no room for difference of opinion in regard to their validity. The first man to prove a certain theorem has not created it; he has discovered an eternal truth. The fact from which he has drawn the veil has not changed its character, it has merely appeared. There was at the beginning of time a body of mathematical truth, dependent on no man's opinion or intelligence, not pragmatic, but absolute, awaiting the coming of a mind which could lay it bare. I say it should be an impressive experience to the young mind when it comes in contact with a new and unimagined truth, and realizes that from everlasting to everlasting that truth must remain the same. Our formulation of the physical laws is modified as we gain new instruments to discover our past inaccuracies, social ideals are purely relative to the time and place in which we may be, but here is something on which we can depend in the midst of the change or decay of our relative knowledge. This experience is of moral value.

The use of the intuition. In the teaching of mathematics it has always been my custom to urge the student to push ahead of his actual demonstration with the imagination. Daily in explaining a new theorem I ask, "How *ought* it to turn out?" In this way the imagination and the intuition are stimulated, and the reasoning faculties come limping along in time, with the demonstration. This, it seems to me, is an important matter. Of course students must use their observation and reasoning, but a sure intuition to break the way for the mind is the rarest and the most delicate gift of all. The powers of observation must be exercised on the facts, and the various features of the problem must be noted; the process of reasoning must be employed on the observed relations and facts. But these processes are of an order far inferior to the keen intuition which can scent the direction in which a truth lies before it is fairly in sight. We must, however, continually emphasize the fact that an accurate intuition can only exist in a mind which is filled with facts which have been mastered, and relations which have been followed out, a mind in which past accomplishment has formed a medium in which the intuition may exist. We observe and reason only because we are finite. Only by this means can we present our results in form which is intelligible to others. A Supreme Intelligence would not need to observe nor to reason; It would know. The feeble approximation to this immediate knowledge which is possible to us is what I mean by the intuition. The great masters of science and mathematics have possessed this gift in large measure. We can all possess it in some measure. And it is our privilege and our duty to stimulate it in our students.

The power to appreciate the unity and harmony of mathematics is closely related to what I have been saying. Mathematics is full of relations which to the mind that is open to them are most suggestive of a unity which extends throughout the whole domain of science. In algebra, the fact that the solution of the quadratic equation affords all the types of number which are needed for all algebraic purposes is not only surprising, but indicates an economy in notation for our subject which is truly remarkable. In geometry the properties of the regular bodies, the sphere, the relations between the sphere and its circumscribing cylinders, and a host of other relations are most suggestive. Why should the square of one side of a *right* triangle exactly equal the sum of the squares of the other sides, that is, the simplest relation numerically be true, for the triangle which is simplest numerical relation be true, for the triangle which is sim-coördination between the realms of number and of form, which ought to be pointed out at every hand. Furthermore, when we pass to the physical world, why is the law of gravitation expressed in the very simple form of $F \backsim m/d^2$? Why should so many of the laws of nature seem to be expressed simply by our purely abstract numerical formulas? One cannot dismiss the question with the statement that we made our numbers to fit the laws, for the formulation of the laws followed centuries later than the adoption of the numerical notation. The numerical laws follow from the natural process of counting, a distinctly human device. The law of gravitation is not determined by our thinking about it, but is independent of our thought. This illustrates what I mean by the unity of science. What is simplest and most beautiful in the domain of pure mathematics, too often corresponds to the facts of nature to be accidental. I contend that it is our privilege to point out at every possible turn this coördination of number and form, of formula and physical law, of unity between the mind and nature. This is an experience of no mean moral value—to realize that our mathematical procedure is attuned to the harmony of the universe.

Columbia University.
New York City.